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HEG-003-0491002 Seat No. _____

B. Sc. / M. Sc. (Applied Physics) (Sem. I) (CBCS) Examination

November / December – 2017

Fundamentals of Mathematics : Paper-II

(New Course)

Faculty Code : 003
Subject Code : 0491002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) All questions are compulsory.
(ii) Numbers in the right margin indicate marks.

1 Attempt any seven short questions : (two marks each) **14**

(1) Define : Symmetric matrix.

(2) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & -1 \end{bmatrix}$ then find AB^T.

(3) Define : Unit matrix.

(4) Find the square root of $7 + 24i$.

(5) Define : Complex number.

(6) Define : Limit and Continuity.

(7) Find $\frac{dy}{dx}$, if $y = x^3 + 3^x + 3^3$.

(8) Evaluate : $\int \left(4x^3 - \frac{1}{x} + \sin x - e^x \right) dx$.

(9) Define : Scalar and Vector.

(10) If $A = (-1, 1, 1)$ and $B = (2, 1, -3)$ then find $[2A + 3B]$.

2 (a) Write answers of any two : (five marks each) **10**

(1) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$ then find non-singular matrices P and Q such that PAQ is in the normal form and hence find rank of A.

(2) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -2 & 4 \\ 1 & 3 & 2 \end{bmatrix}$.

(3) Solve $x + y + 2z = 9$; $2x + 4y - 3z = 1$; $3x + 6y - 5z = 0$ by matrix method.

(4) Define : Orthogonal matrix. Verify if the matrix $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & 1 \end{bmatrix}$ is an orthogonal matrix.

(b) Write answers of any two : (two marks each) **4**

(1) Expand : $\begin{vmatrix} 3 & -1 & 2 \\ 1 & 0 & 6 \\ -2 & 5 & 4 \end{vmatrix}$.

(2) Define : Adjoint of a matrix.

(3) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$ then find AB^{-1}.

(4) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ then find A^2.

3 (a) Write answers of any two : (five marks each) **10**

(1) State and prove De Moivre's theorem.

(2) Express the following complex numbers in polar form. Also find modulus and principal argument.

(a) $1 + \sqrt{3}i$

(b) $\frac{1+i}{1-i}$

(3) If $(x+iy)^3 = a+ib$ then prove that

$$\frac{a}{x}+\frac{y}{b}=4\left(x^2-y^2\right).$$

(4) Prove that $\left(\frac{\cos\theta+i\sin\theta}{\sin\theta+i\cos\theta}\right)^4 = \cos 8\theta + i\sin 8\theta.$

(b) Write answers of any two : (two marks each) **4**

(1) Express $(2-3i)(-2+i)$ in the form of $x+iy,\ x,\ y \in R.$

(2) Find the conjugate complex number and modulus of $\frac{-3+7i}{1+i}.$

(3) Find the cube root of unity.

(4) Obtain $x, y \in R$ from the $x+4iy = xi+y+3.$

4 (a) Write answers of any two : (five marks each) **10**

(1) Evaluate :

(a) $\lim\limits_{x\to 0}\ \frac{e^x+\sin x-1}{x}$

(b) $\lim\limits_{x\to\infty}\ \frac{x(x+1)}{x^2+5x+6}.$

(2) Find $\frac{dy}{dx}$, if

(a) $y=\log(\sec x+\tan x)$ and

(b) $y=\cos(\log x).$

(3) Evaluate using reduction formula.

(a) $\int \cos^5 x\,dx$

(b) $\int \tan^6(2x)\,dx$

(4) Evaluate : $\int_3^8 \frac{x}{(x-1)(x-2)}\,dx.$

(b) Write answers of any two : (two marks each) **4**

(1) Define : Odd and even function.

(2) Evaluate : $\lim_{x \to 1} \frac{x^2 - 4x + 3}{x^2 + 2x - 3}$.

(3) Find $\frac{dy}{dx}$, if $y = x^7 e^x \log x$.

(4) Evaluate : $\int xe^x dx$.

5 (a) Write answers of any two : (five marks each) **10**

(1) Simplify :

$(10i + 2j + 3k) \cdot [(i - 2j + 2k) \times (3i - 2j - 2k)]$.

(2) Prove that angle between two vectors $3i + j + 2k$ and $2i - 2j + 4k$ is $\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$.

(3) If $a = 2i - 3j + 4k$ and $b = i - j + k$ then find perpendicular unit vector to both $a + b$ and $a - b$.

(4) Find curl, divergence and gradient at $P(1, -2, 1)$ for $f(x) = 3x^2 y - y^3 z^2$.

(b) Write answers of any two : (two marks each) **4**

(1) Find the unit vector in the direction of the vector $(2, -1, 3)$.

(2) If $a = 2i + j - 3k$, $b = 4i + 5j + 4k$ and $c = 3i - 2j + k$ then find $3a + 2b - 3c$.

(3) If $a = 3i - j - 4k$, $b = -2i + 4j - 3k$ and $c = 2j - i - 5k$ then find direction cosines of $a + 2b - c$.

(4) If $a = (2, -3, 5)$, $b = (x, -6, -8)$ and $a.b = 0$ then find the value of x.